Exercise 36

Find the critical numbers of the function.

$$h(p) = \frac{p-1}{p^2+4}$$

Solution

A critical number is a value of p for which the derivative is zero or nonexistent. Take the derivative of the function.

$$\begin{aligned} h'(p) &= \frac{d}{dp}\left(\frac{p-1}{p^2+4}\right) \\ &= \frac{\left[\dfrac{d}{dp}(p-1)\right](p^2+4) - \left[\dfrac{d}{dp}(p^2+4)\right](p-1)}{(p^2+4)^2} \\ &= \frac{(1)(p^2+4) - (2p)(p-1)}{(p^2+4)^2} \\ &= \frac{(p^2+4) - (2p^2-2p)}{(p^2+4)^2} \\ &= \frac{-p^2+2p+4}{(p^2+4)^2} \end{aligned}$$

Set what's in the numerator and denominator equal to zero and solve for p.

$$-p^2+2p+4=0 \qquad (p^2+4)^2=0$$

$$p = \frac{-2 \pm \sqrt{2^2-4(-1)(4)}}{2(-1)} \qquad p^2+4=0$$

$$p = \frac{-2 \pm \sqrt{20}}{-2} \qquad p^2=-4$$

$$p = \frac{-2 \pm 2\sqrt{5}}{-2} \qquad p = \sqrt{-4} \quad \text{or} \quad p = -\sqrt{-4}$$

$$p = 1 \mp \sqrt{5} \qquad p = 2i \quad \text{or} \quad p = -2i$$

Since only real numbers can be critical numbers of the function,

$$p = 1 - \sqrt{5} \quad \text{or} \quad p = 1 + \sqrt{5}.$$